ON THE STEADY FLOW OF A VISCOUS ELECTRICALLY CONDUCTING GAS IN A PLANE CHANNEL IN THE PRESENCE OF TRANSVERSE FIELDS

(O STATSIONARNOM TECHENII VIAZKOGO ELEKTROPROVODNOGO GAZA V PLOSKOM KANALE PRI NALICHII SKRESHCHENNYKH POLEI)

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In studying flows of electrically conducting fluids in channels of various configurations, it is significant to take account of the compressibility and also of the dependence of the transport coefficients on the variables of state of an ionized gas. Using somewhat broad assumptions in regard to the properties of the gas Bleviss [1] obtained and investigated a solution for the magnetogasdynamic Couette flow. Calculations of the dependence of the conductivity upon the temperature lead to the determination of the hysteresis character of the variation of the wall friction. These findings were confirmed later by Bush [2] who investigated the compressibility of a magnetogasdynamical boundary layer on a plate.

In the present paper it will be shown that a solution of the problem of steady flow of an ionized gas in a plane channel may be obtained for an arbitrarily chosen law of variation with temperature of the coefficients of viscosity and of electroconductivity when the motion is caused by external mutually normal magnetic and electrical fields.

Consider the steady motion of an electroconductive gas in an infinitely long plane channel in the presence of mutually normal electric and magnetic fields. Let us choose a right handed coordinate system so that

$$v_x = u(y), \quad B_y = B_0, \quad E_z = -E_0, \quad j_z = j(y)$$

Because all the quantities in a given case depend only on the transverse coordinate y, the initial system of equations has the form

$$\frac{d}{dy}\left(\eta\frac{du}{dy}\right)=jB_0, \qquad \frac{dp}{dy}=jB_x, \qquad p=R\rho T$$
$$\frac{d}{dy}\left[\eta\frac{d}{dy}\left(\frac{u^2}{2}+\frac{c_pT}{P}\right)\right]=jE_0, \qquad j=-\frac{dH_x}{dy}=\sigma(B_0u-E_0) \qquad \left(P=\frac{\eta c_p}{\lambda}\right) \tag{1}$$

where the Prandtl number P and the specific heat c_p are considered to be constant. η and λ are the coefficients of viscosity and of heat conductivity, respectively. After eliminating the current density j from the equations of motion and of energy, we obtain

$$\frac{d}{dy}\left[\eta\frac{d}{dy}\left(\frac{u^2}{2}+\frac{c_pT}{P}-\frac{E_0}{B_0}u\right)\right]=0 \tag{2}$$

By integration of this equation with the boundary conditions $u = 0$ and $T = T_w$ for $y = \pm a$, (where $2a$ is the height of the channel, T_w is the temperature of the walls) we find the following relation between the temperature and the velocity of the gas:

$$T=T_w+\frac{P}{c_p}\left(\frac{E_0}{B_0}u-\frac{u^2}{2}\right) \tag{3}$$

Eliminating now the current density from the equations of motion and Ohm's law, choosing the gas velocity to be the new variable and introducing the laminar shear stress $\tau = \eta du/dy$ we obtain

$$\tau\frac{d\tau}{du}=B_0^2\sigma\eta(u-u_0) \qquad \left(u_0=\frac{E_0}{B_0}\right) \tag{4}$$

In integrating Equation (4) a constant of integration is determined from the boundary condition

$$\tau=0,\ u=u_m \quad \text{for } y=0 \tag{5}$$

where u_m is the unknown maximum velocity occurring on the axis of the pipe.

We obtain the relation between the friction and the velocity

$$\tau=\eta\frac{du}{dy}=\mp J(u,u_m),\ J(u,u_m)=\left(2B_0^2\int_u^{u_m}\sigma(u)\eta(u)(u_0-u)\,du\right)^{1/2} \tag{6}$$

where the upper sign should be assumed for $y > 0$ and the lower for $y < 0$. Since the coefficients σ and η are assumed to be given functions of the temperature, then on the basis of (3) they may be considered to be known functions of the velocity.

Integration of Equation (6) gives the desired relation between the velocity of the gas and the coordinate

$$y = \pm\left(a - \int_0^u \frac{\eta(u)\,du}{J(u, u_m)}\right) \tag{7}$$

If herein it is assumed that for $y = 0$, $u = u_m$, we obtain from it a relation either for the determination of u_m for a given dimension a of the channel or for the determination of dimension a for a given maximum gas velocity u_m

$$a = \int_0^{u_m} \frac{\eta(u)\,du}{J(u, u_m)} \tag{8}$$

Using (8), we may write (7) in the form

$$y = \pm \int_u^{u_m} \frac{\eta(u)\,du}{J(u, u_m)} \tag{9}$$

For particular values of constants η and σ the evaluation of the integral in (8) yields

$$u_m = u_0\left(1 - \frac{1}{\cosh M}\right) \text{ where } \left(M = B_0 a \sqrt{\frac{\sigma}{\eta}}\right) \text{ —is the Hartman number} \tag{10}$$

When completing the integration of (9) and taking into account (10), we find the following equation for the distribution of velocities in a channel:

$$u = u_0\left(1 - \frac{\cosh My/a}{\cosh M}\right) \tag{11}$$

It is not difficult to generalize the solution obtained in [1] of variable specific heat capacity c_p.

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